The University of Tennessee at Martin

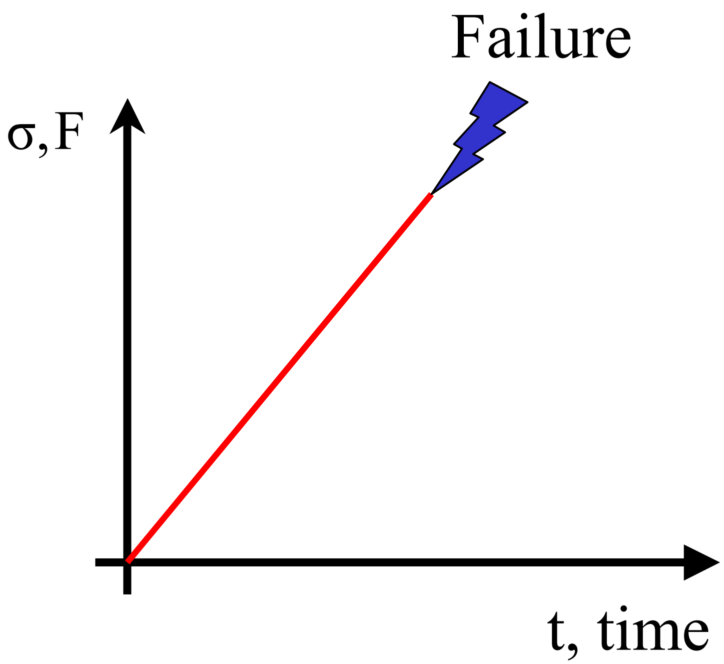
MARTIN School of Engineering

Fatigue

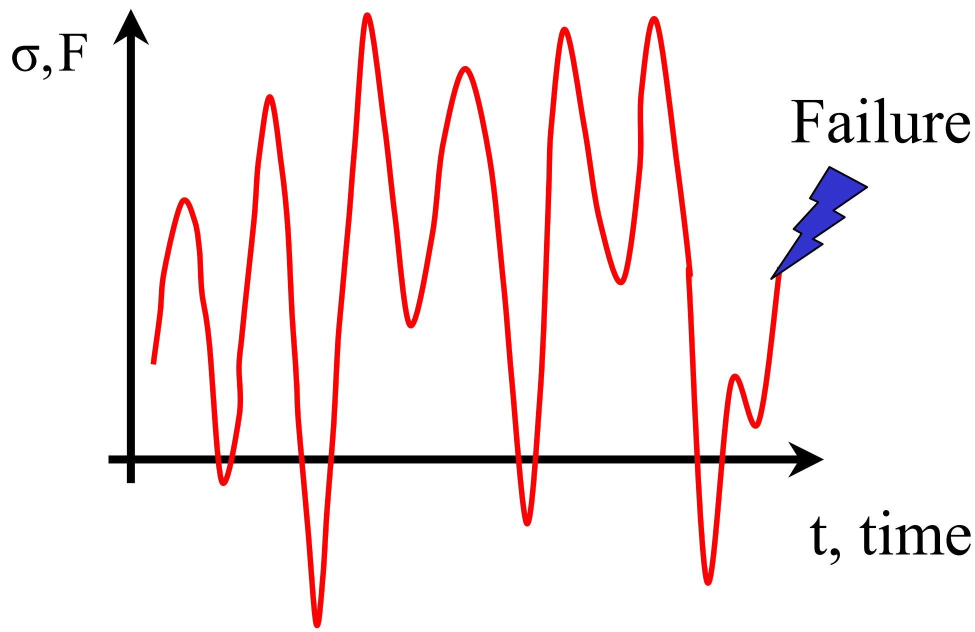
Lecture 10

Engineering 473
Machine Design

Load Histories and Design Objectives



Monotonic, Static, or Steady

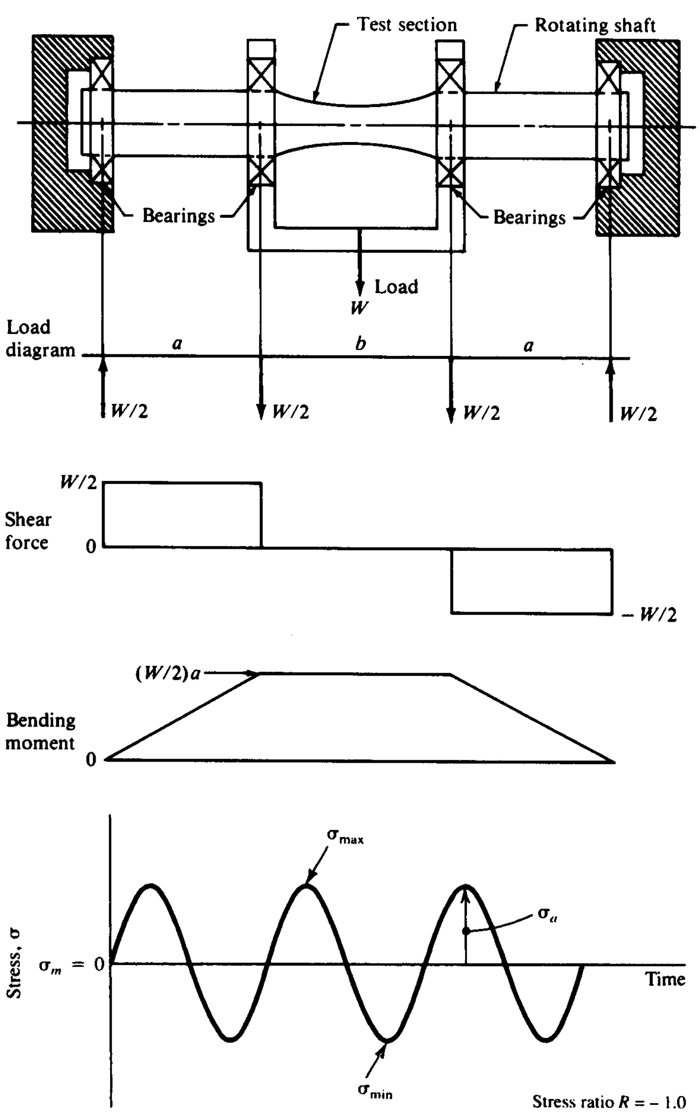


Dynamic, Cyclic, or Unsteady

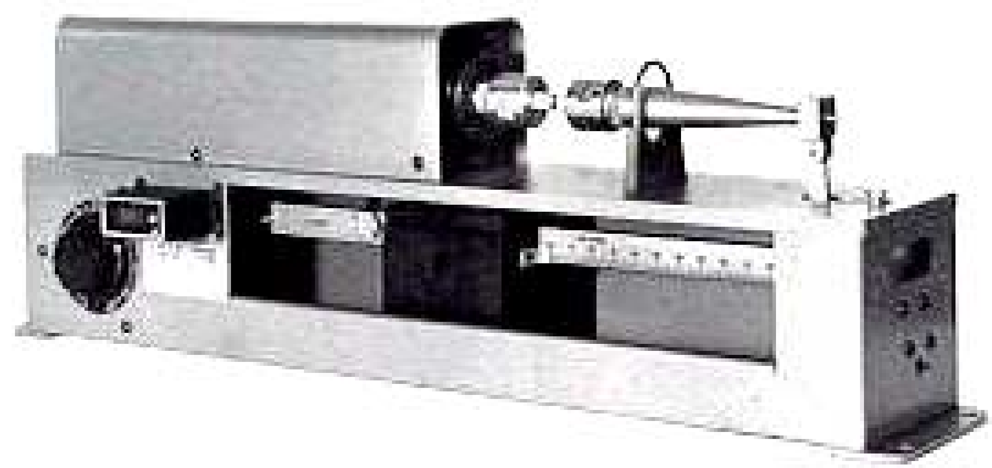
Design for Strength

Design for Life

Rotating Beam Fatigue Testing



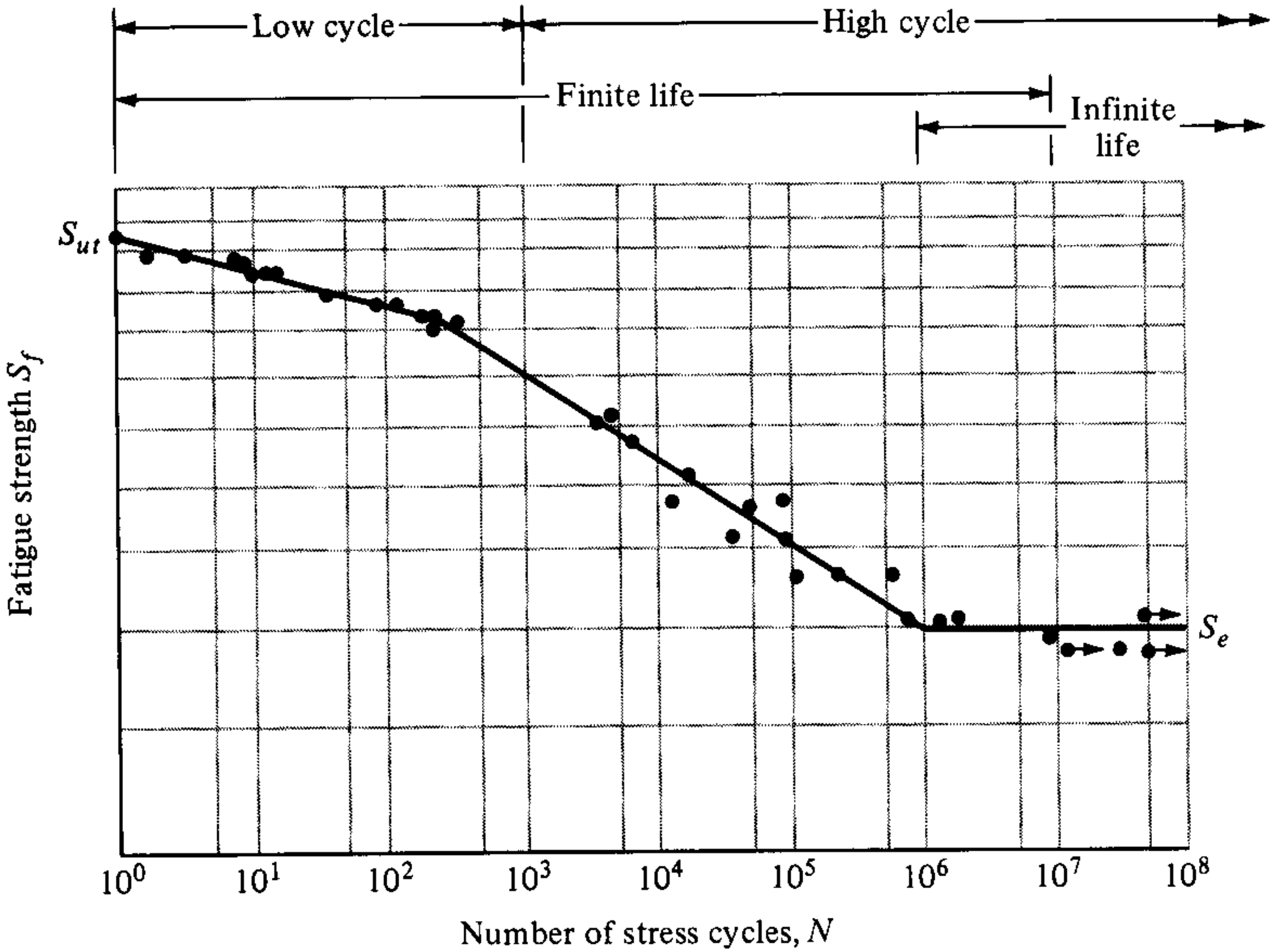
Mott, Fig. 5-2 & 5-3



Fatigue Dynamics, Inc. rotating beam test equipment.

www.fdinc.com

S-N Curve



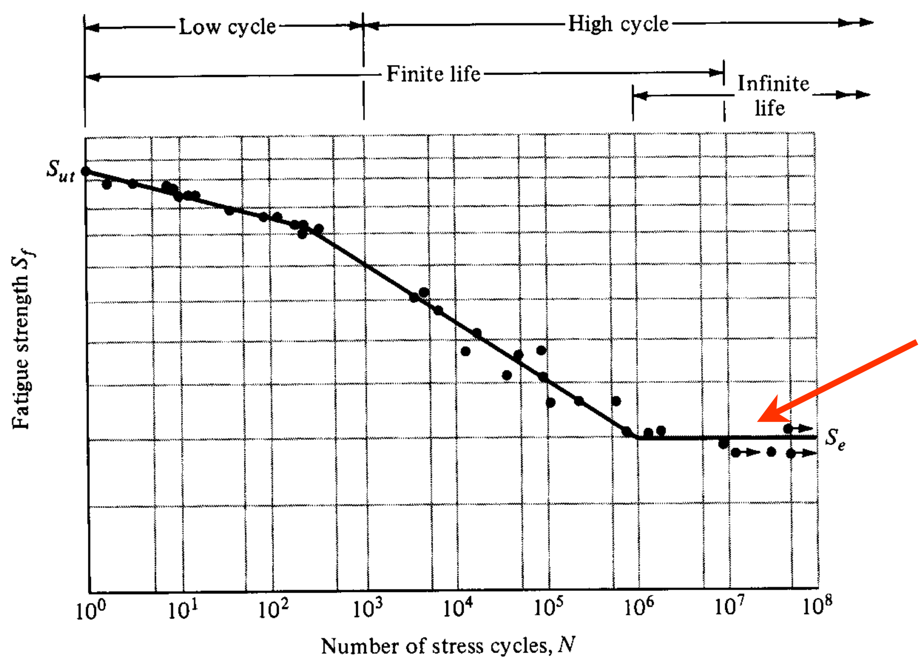
Completely reversed cyclic stress, UNS G41200 steel

Shigley, Fig. 7-6

Fatigue Strength

The **Fatigue Strength**, **$S_f(N)$**, is the stress level that a material can endure for N cycles.

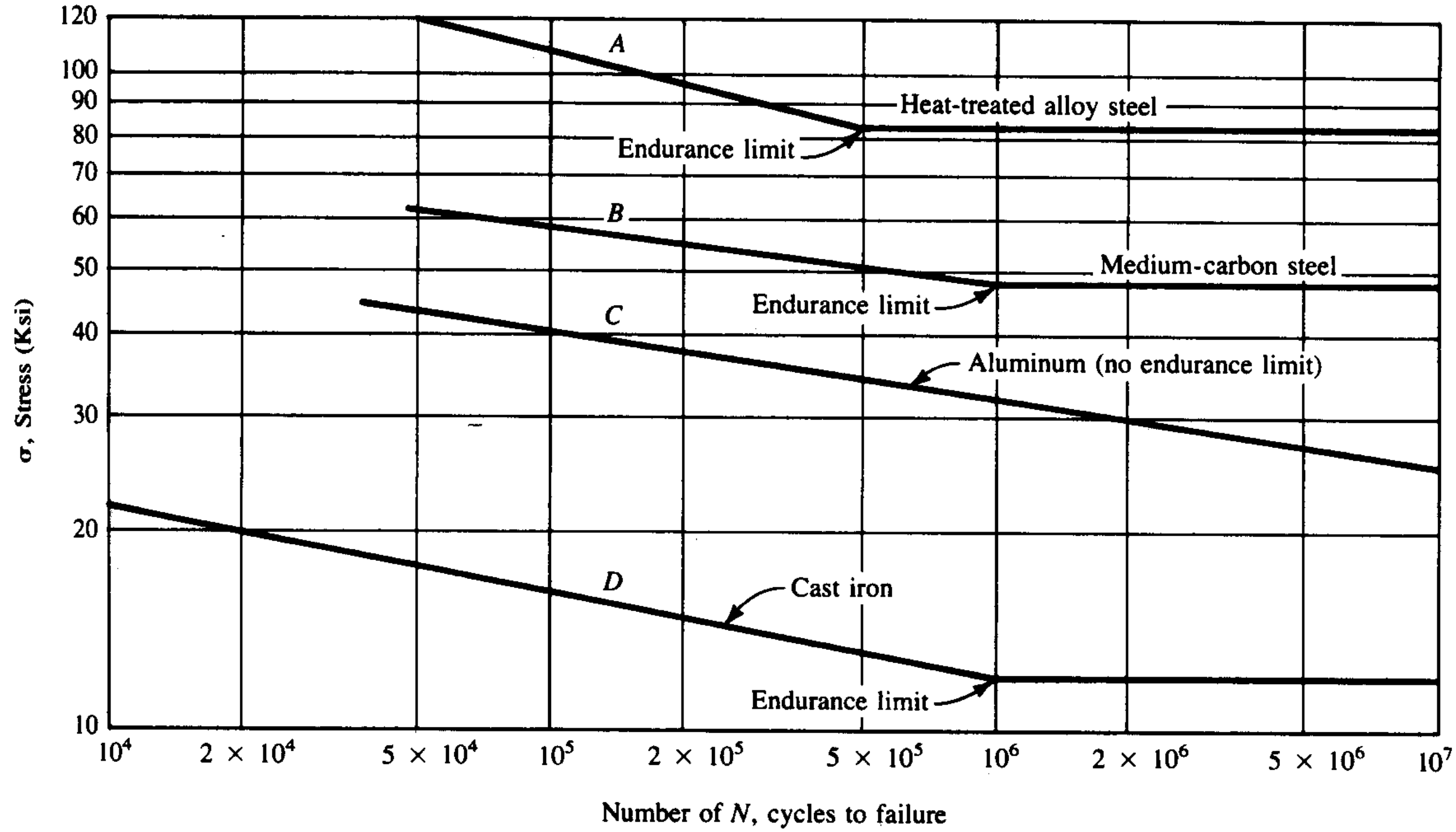
The stress level at which the material can withstand an infinite number of cycles is call the **Endurance Limit**.



The Endurance Limit is observed as a horizontal line on the S-N curve.

Shigley, Fig. 7-6

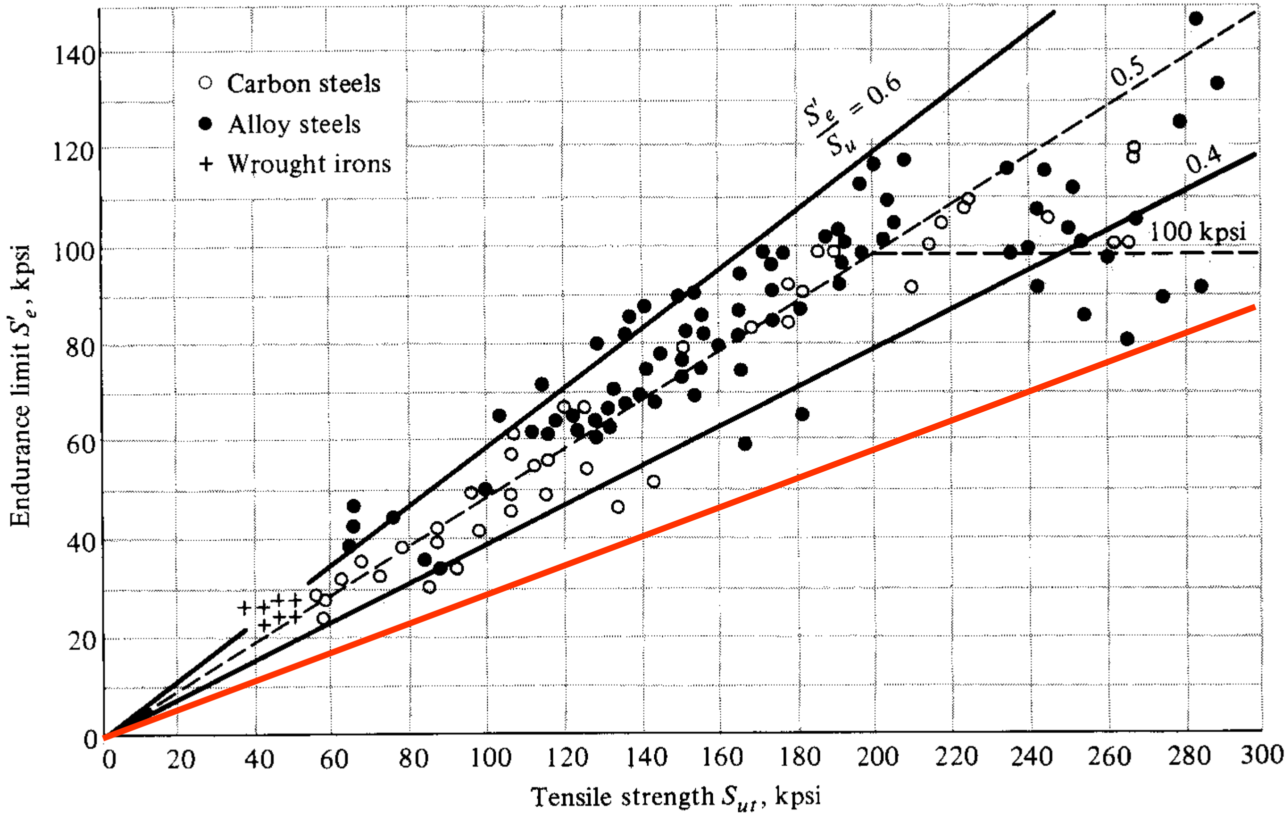
Representative S-N Curves



Note that non-ferrous materials often exhibit no endurance limit.

Mott, Fig. 5-7

Endurance Limit Vs Tensile Strength



Conservative Lower Bound for Ferrous Materials

$S'_e = 0.3S_{ut}$

$S'_e \equiv$ Endurance Limit of Test Specimen

$S_{ut} \equiv$ Tensile Strength of Test Specimen

Shigley, Fig. 7-7

Endurance Limit Multiplying Factors

(Marin Factors)

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S'_e$$

$S_e \equiv$ Endurance limit of part

$S'_e \equiv$ Endurance limit of test specimen

$k_a \equiv$ Surface factor

$k_b \equiv$ Size factor

$k_c \equiv$ Load factor

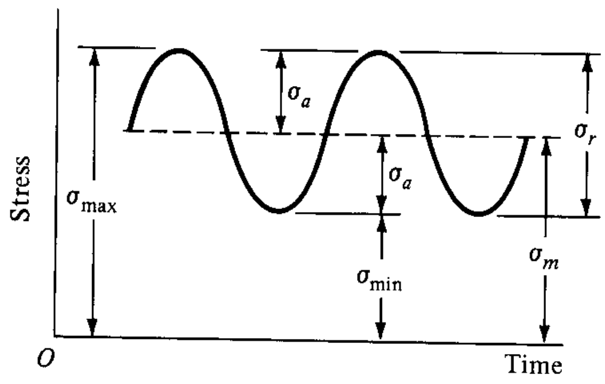
$k_d \equiv$ Temperature factor

$k_e \equiv$ Miscellaneous - effects factor

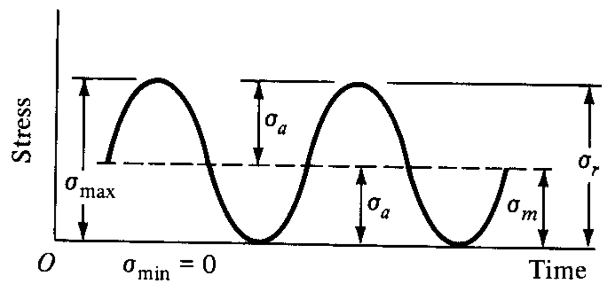
There are several factors that are known to result in differences between the endurance limits in test specimens and those found in machine elements.

See sections 7-8 & 7-9 in Shigley for a discussion on each factor.

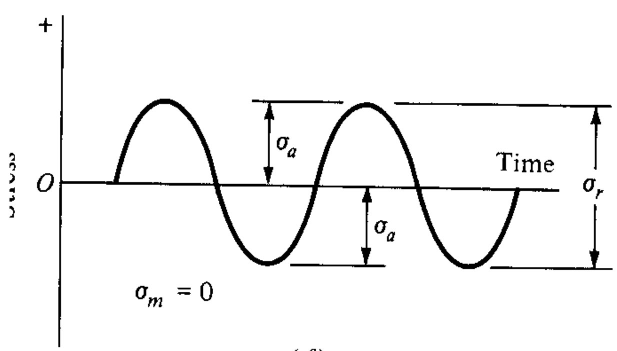
Mean Stress Effects



(d)



(e)

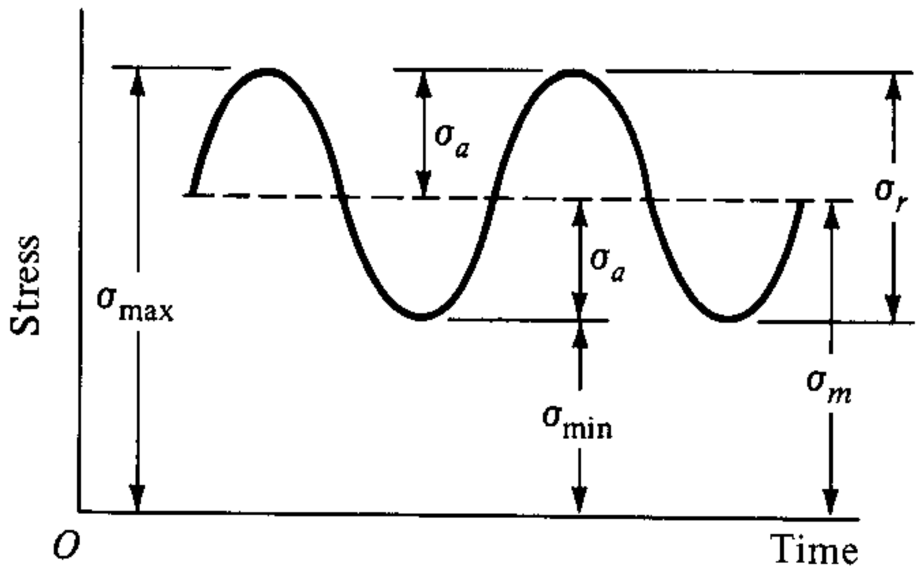


(f)

- The S-N curve obtained from a rotating beam test has completely reversed stress states.
- Many stress histories will not have completely reversed stress states.

Shigley, Fig. 7-12

Definitions



Stress Range

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

Alternating Stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Mean Stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Stress Ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Amplitude Ratio

$$A = \frac{\sigma_a}{\sigma_m}$$

Note that R=-1 for a completely reversed stress state with zero mean stress.

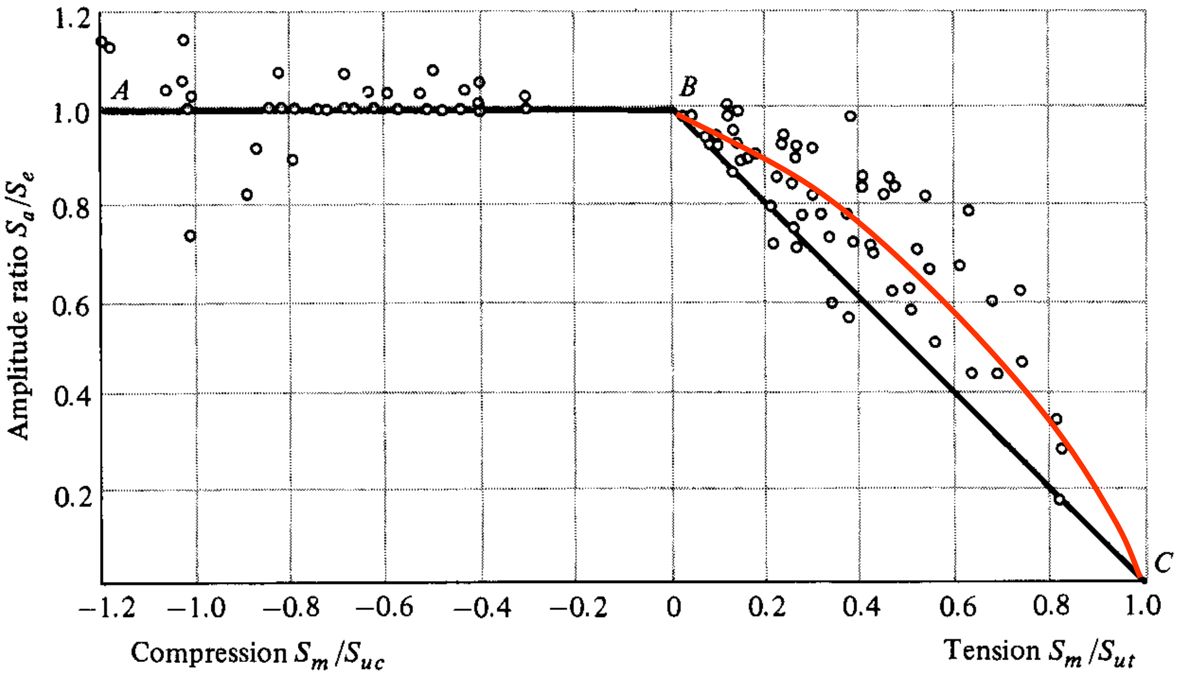
Mean Stress Fatigue Testing

SC-6 Shutdown Controller.

Fatigue Dynamics, Inc., fluctuating fatigue stress testing equipment.

www.fdinc.com

Fluctuating Stress Failure Data



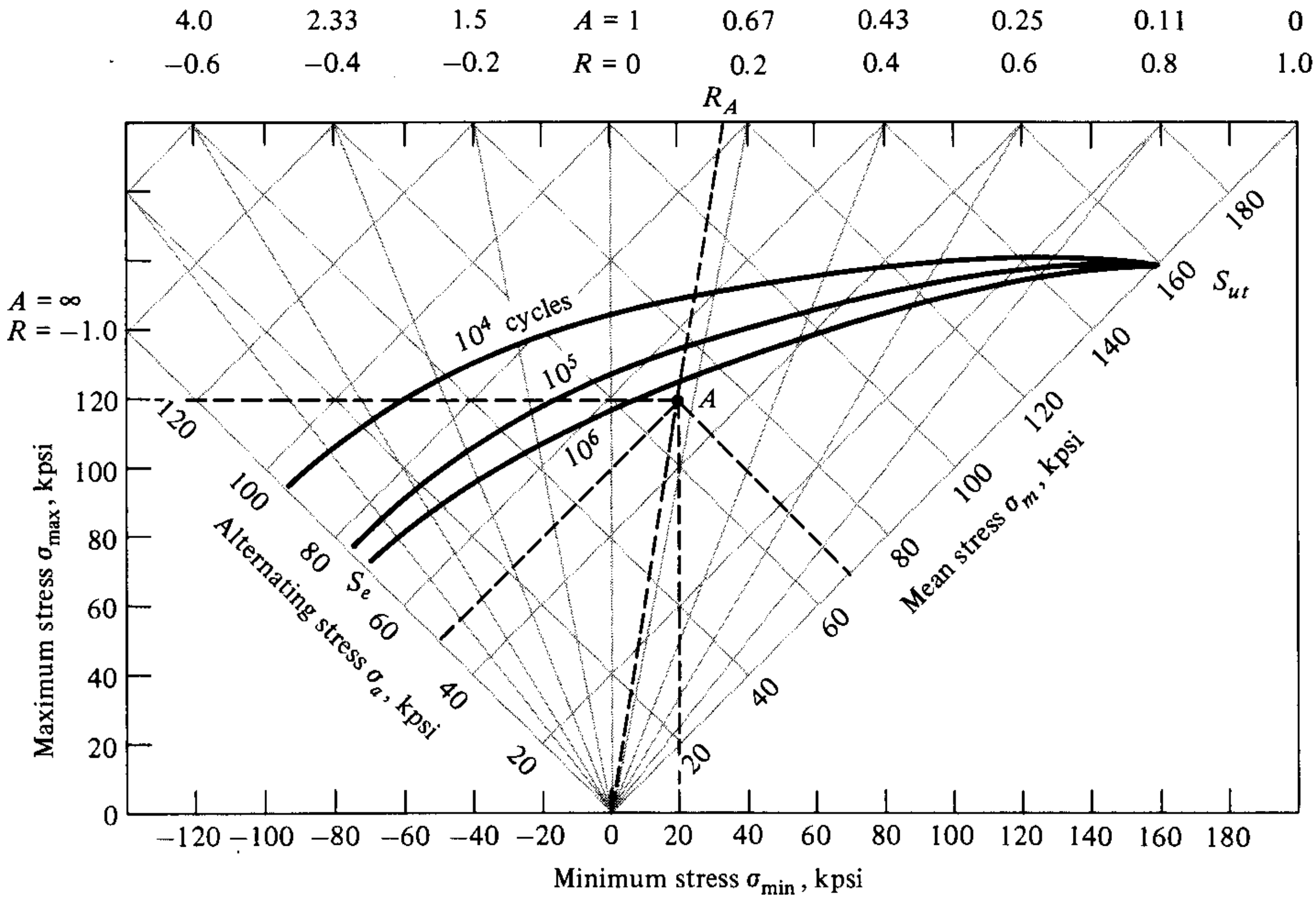
Note that a tensile mean stress results in a significantly lower fatigue strength for a given number of cycles to failure.

This plot shows the fatigue strength of several steels as a function of mean stress for a constant number of cycles to failure.

Note that a curved line passes through the mean of the data.

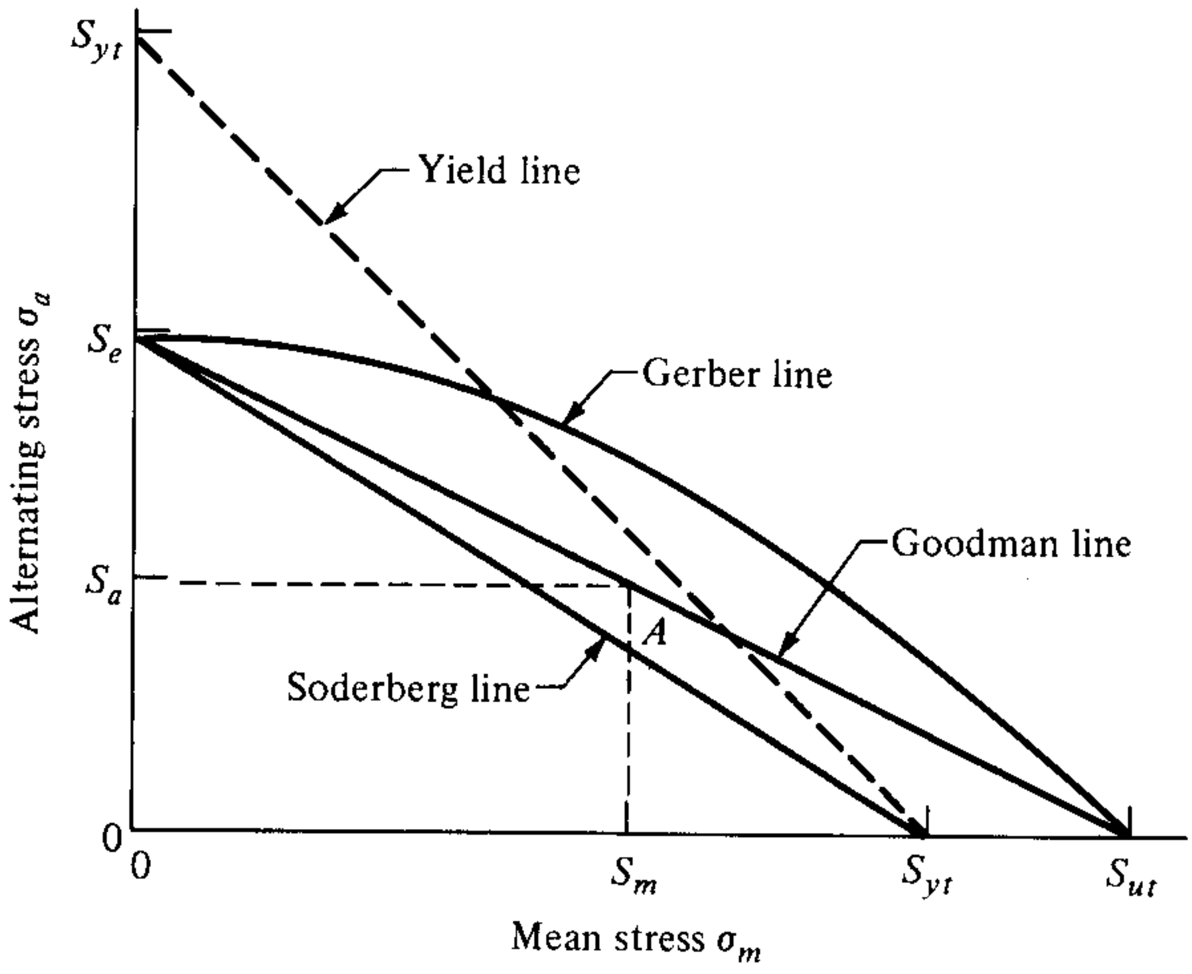
Shigley, Fig. 7-14

Master Fatigue Plot



Shigley, Fig. 7-15

Fluctuating Stress Failure Interaction Curves

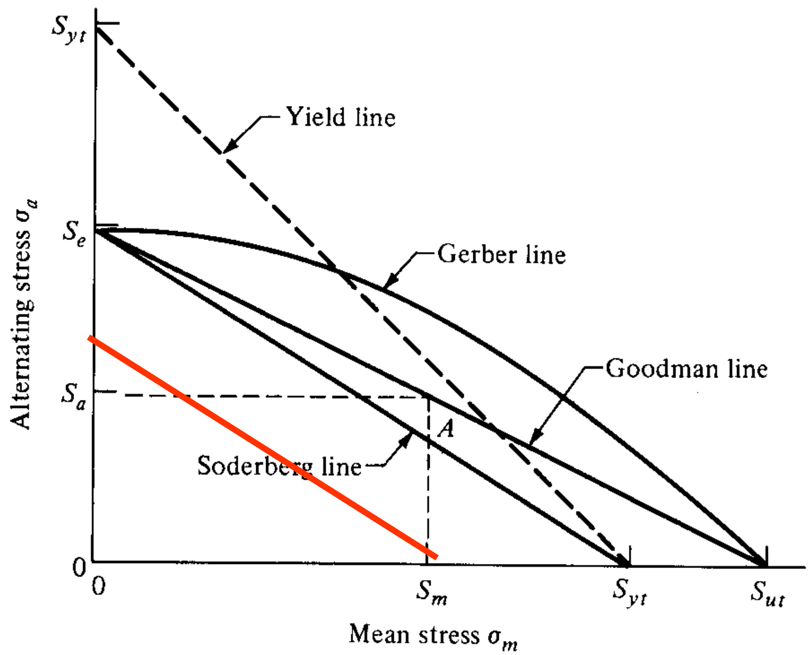


Shigley, Fig. 7-16

Soderberg Interaction Line

$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{yt}} = 1$$

Any combination of mean and alternating stress that lies on or below the Solderberg line will have infinite life.



Factor of Safety Format

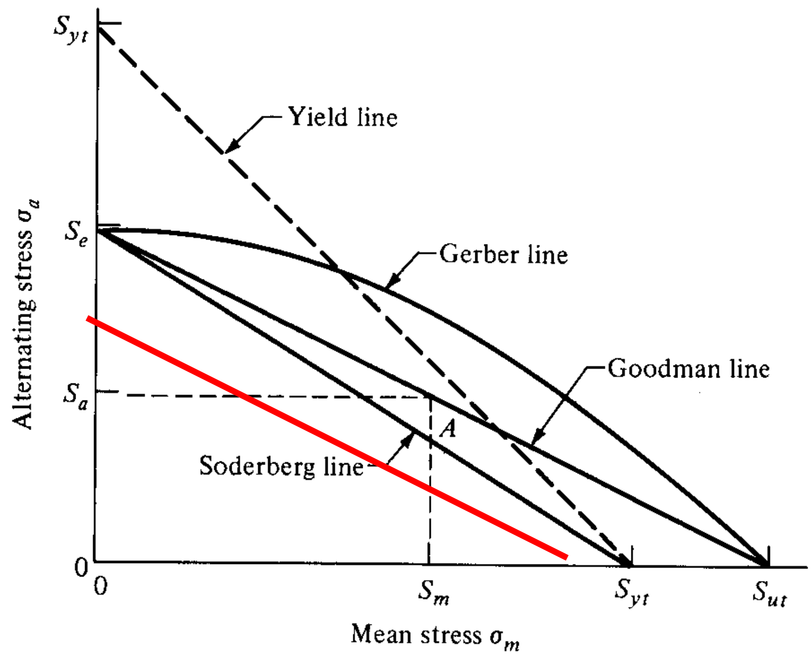
$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{yt}} = \frac{1}{N_f}$$

Note that the fatigue stress concentration factor is applied only to the alternating component.

Goodman Interaction Line

$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

Any combination of mean and alternating stress that lies on or below the Goodman line will have infinite life.



Factor of Safety Format

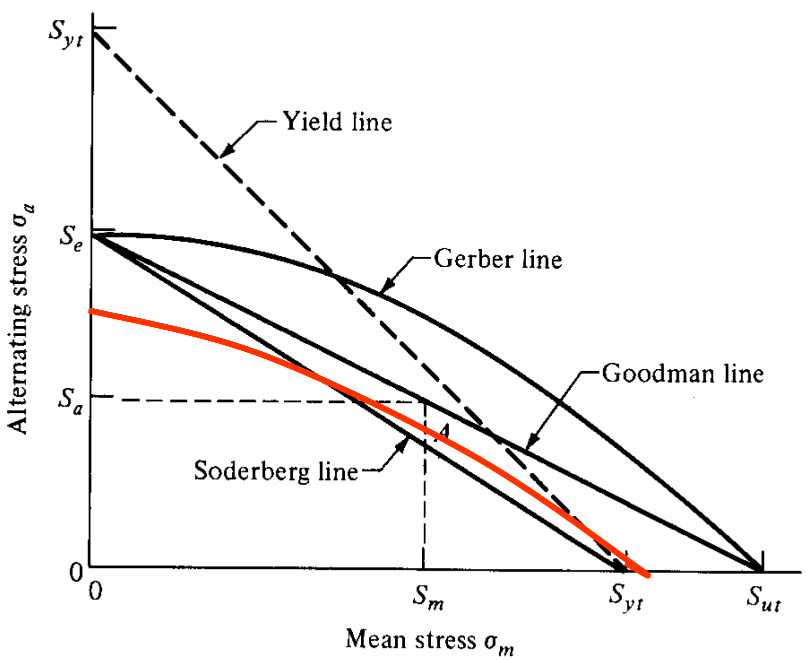
$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{ut}} = \frac{1}{N_f}$$

Note that the fatigue stress concentration factor is applied only to the alternating component.

Gerber Interaction Line

$$\frac{k_f S_a}{S_e} + \left(\frac{S_m}{S_{ut}} \right)^2 = 1$$

Any combination of mean and alternating stress that lies on or below the Gerber line will have infinite life.

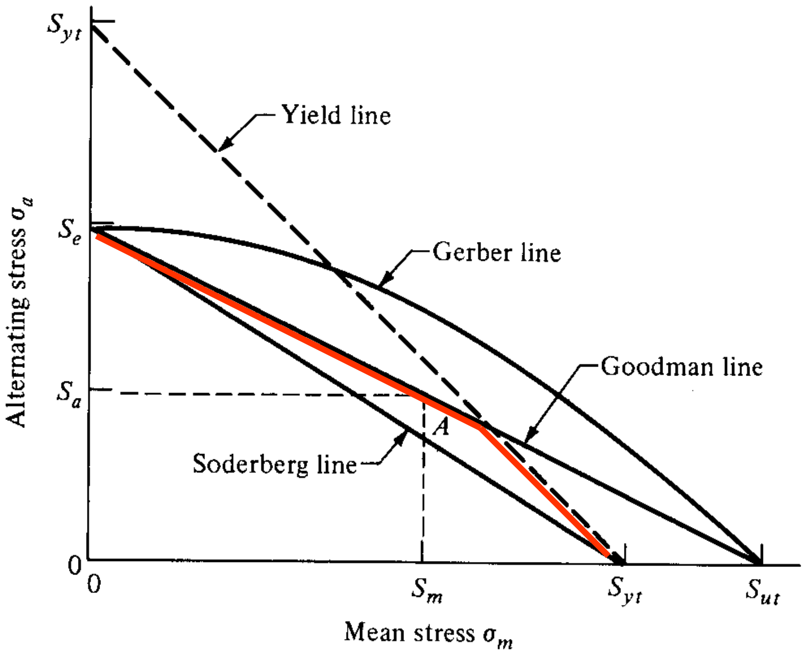


Factor of Safety Format

$$\frac{k_f N_f S_a}{S_e} + \left(\frac{N_f S_m}{S_{ut}} \right)^2 = 1$$

Note that the fatigue stress concentration factor is applied only to the alternating component.

Modified-Goodman Interaction Line



The Modified-Goodman Interaction Line never exceeds the yield line.

Example No. 1

A 1.5-inch round bar has been machined from AISI 1050 cold-drawn round bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the design of the ends and the fillet radius, a fatigue stress-concentration factor of 1.85 exists. The remaining Marin factors have been worked out, and are k_a=0.797, k_b=k_d=1, and k_c=0.923. Find the factor of safety using the Goodman interaction line.

Shigley, Example 7-5

Example No. 1
(Continued)

$$S_{ut} = 100.\text{ ksi}$$
$$S'_e \approx 0.50 \cdot S_{ut} = 50.\text{ ksi}$$

$$A = \frac{\pi \cdot d^2}{4} = 1.77\text{ in}^2$$

$$\sigma_{max} = \frac{16\text{ kip}}{1.77\text{ in}^2} = 9.04\text{ ksi}$$
$$\sigma_{min} = 0\text{ ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 4.52\text{ ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 4.52\text{ ksi}$$

$$S_e = k_a k_b k_c k_d S'_e$$
$$= (0.797)(1)(0.923)(1)(50\text{ ksi})$$
$$S_e = 36.8\text{ ksi}$$

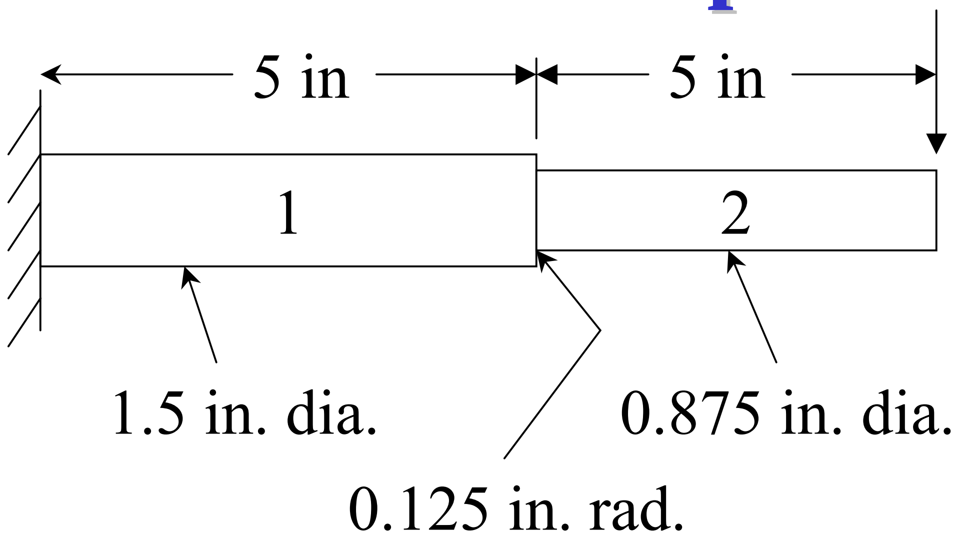
Example No. 1
(Continued)

$$\frac{k_f \sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{N_f}$$

$$\frac{1.85 \cdot 4.52\,\text{ksi}}{36.8\,\text{ksi}} + \frac{4.52\,\text{ksi}}{100.\,\text{ksi}} = 0.272 = \frac{1}{N_f}$$

$$\boxed{N_f = 3.67}$$

Example



$P_{max} = 1000 \text{ lb}$

$P_{min} = 350 \text{ lb}$

Material UNS G41200 Steel

Notch sensitivity q=0.3

Will the beam have infinite life?

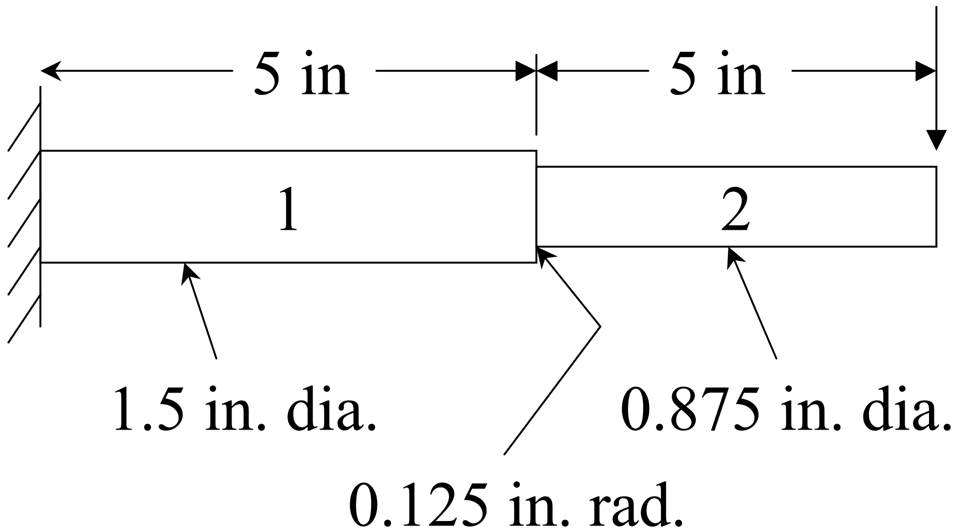
$$I_1 = \frac{\pi}{64} D_1^4 = \frac{\pi}{64}(1.5)^4 = 0.249 \text{ in}^4$$

$$I_2 = \frac{\pi}{64} D_2^4 = \frac{\pi}{64}(0.875)^4 = 0.088 \text{ in}^4$$

$$S_1 = \frac{I_1}{c_1} = \frac{0.249 \text{ in}^4}{0.75 \text{ in}} = 0.332 \text{ in}^3$$

$$S_2 = \frac{I_2}{c_2} = \frac{0.088 \text{ in}^4}{0.438 \text{ in}} = 0.201 \text{ in}^3$$

Example
(Continued)



$P_{max} = 1000 \text{ lb}$

$P_{min} = 350 \text{ lb}$

Material UNS G41200 Steel

Notch sensitivity q=0.3

$$q = \frac{k_f - 1}{k_t - 1}$$

$$k_f = 1 + q(k_t - 1)$$

$$\frac{D}{d} = \frac{1.5 \text{ in}}{0.875 \text{ in}} = 1.71$$

$$\frac{r}{d} = \frac{0.125}{0.875} = 0.143$$

Ref. Peterson

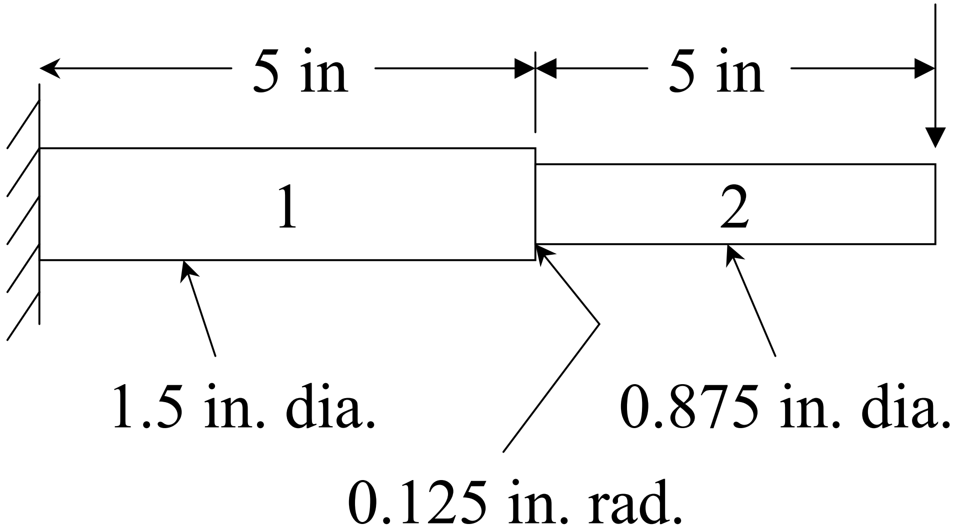
$$k_t = 1.61$$

$$k_f = 1 + q(k_t - 1)$$

$$= 1 + 0.3(1.61 - 1)$$

$$= 1.18$$

Example
(Continued)



$P_{max} = 1000\,\text{lb}$

$P_{min} = 350\,\text{lb}$

Material UNS G41200 Steel

Notch sensitivity q=0.3

Section 1 (Base)

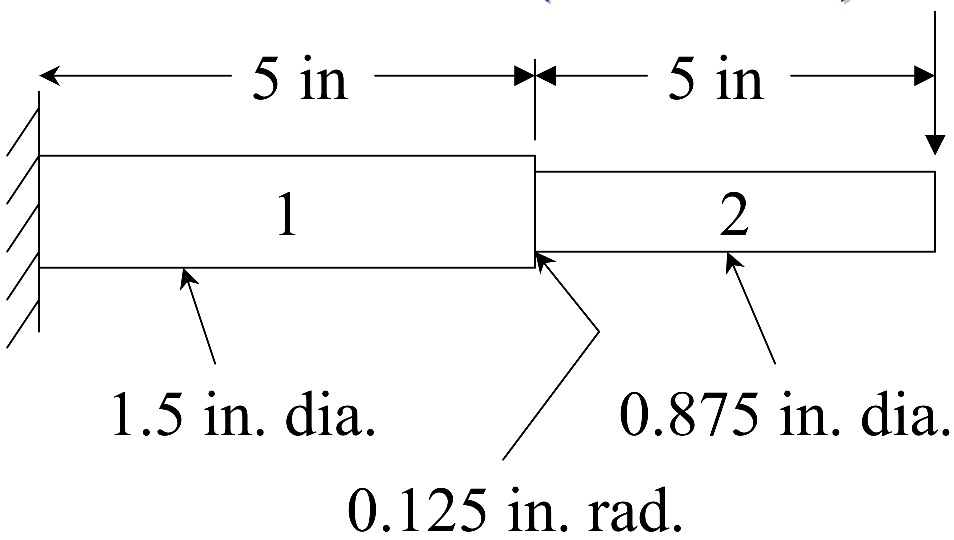
$$\sigma_{max} = \frac{M_1}{S_1} = \frac{(1000\,\text{lb})(10\,\text{in})}{0.332\,\text{in}^3} = 30.1\,\text{ksi}$$

$$\sigma_{min} = \frac{M_1}{S_1} = \frac{(350\,\text{lb})(10\,\text{in})}{0.332\,\text{in}^3} = 10.5\,\text{ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 9.8\,\text{ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 20.3\,\text{ksi}$$

Example
(Continued)



$$P_{max} = 1000\,\text{lb}$$

$$P_{min} = 350\,\text{lb}$$

Material UNS G41200 Steel
Notch sensitivity q=0.3

Section 2 (Fillet)

$$\sigma_{max} = \frac{M_1}{S_1} = \frac{(1000\,\text{lb})(5\,\text{in})}{0.201\,\text{in}^3} = 24.9\,\text{ksi}$$

$$\sigma_{min} = \frac{M_1}{S_1} = \frac{(350\,\text{lb})(5\,\text{in})}{0.201\,\text{in}^3} = 8.71\,\text{ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 8.10\,\text{ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 16.8\,\text{ksi}$$

Example
(Continued)

Section 1 (Base)

$$\sigma_{max} = \frac{M_1}{S_1} = \frac{(1000\ \text{lb})(10\ \text{in})}{0.332\ \text{in}^3} = 30.1\ \text{ksi}$$

$$\sigma_{min} = \frac{M_1}{S_1} = \frac{(350\ \text{lb})(10\ \text{in})}{0.332\ \text{in}^3} = 10.5\ \text{ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 9.8\ \text{ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 20.3\ \text{ksi}$$

$$S_{ut} = 116\ \text{ksi}$$

$$S'_e = 30\ \text{ksi} = S_e$$

$$\frac{k_f \sigma_a}{S_e} + \frac{\sigma_m}{S_{ult}} = \frac{1}{N_f}$$

$$\frac{1.0(9.8\ \text{ksi})}{30\ \text{ksi}} + \frac{20.3\ \text{ksi}}{116\ \text{ksi}} = 0.502$$

$$N_f = \frac{1}{0.502} = 1.99$$

Part has infinite life.

Example
(Continued)

Section 2 (Fillet)

$$\sigma_{max} = \frac{M_1}{S_1} = \frac{(1000\ \text{lb})(5\ \text{in})}{0.201\ \text{in}^3} = 24.9\ \text{ksi}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 8.10\ \text{ksi}$$

$$\sigma_{min} = \frac{M_1}{S_1} = \frac{(350\ \text{lb})(5\ \text{in})}{0.201\ \text{in}^3} = 8.71\ \text{ksi}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 16.8\ \text{ksi}$$

$$S_{ut} = 116\ \text{ksi}$$

$$S'_e = 30\ \text{ksi} = S_e$$

$$\frac{k_f \sigma_a}{S_e} + \frac{\sigma_m}{S_{ult}} = \frac{1}{N_f}$$

$$\frac{1.18(8.10\ \text{ksi})}{30\ \text{ksi}} + \frac{16.8\ \text{ksi}}{116\ \text{ksi}} = 0.463$$

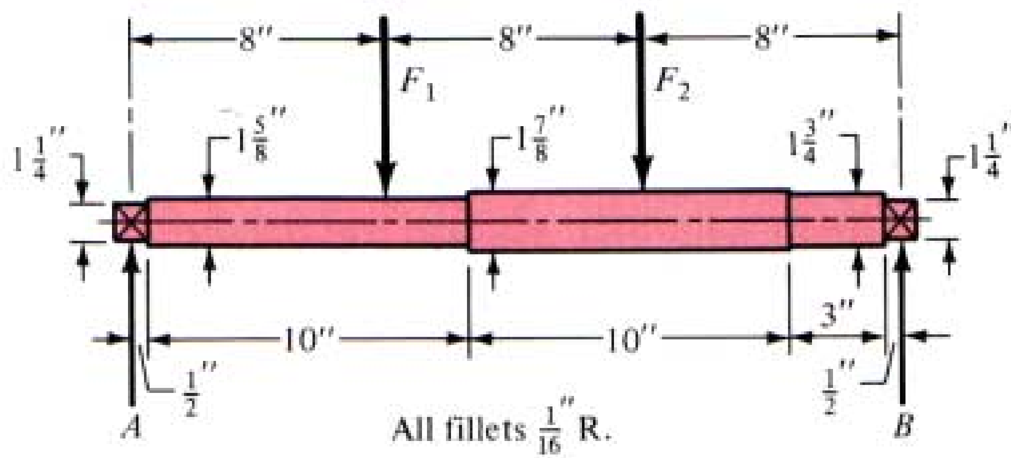
$$N_f = \frac{1}{0.463} = 2.16$$

Part has infinite life.

Assignment

Problem 1

The steel shaft shown in the figure has a ground finish and a minimum tensile strength of 89 kpsi. The shaft rotates at 1720 rev/min and is supported in rolling bearings at A and B. Estimate the life of the part if the forces are $F_1 = 2000$ lb and $F_2 = 3000$ lb.



Assignment
(Continued)

Problem 2

The flat steel spring illustrated is loaded in bending by the force F. The spring supports a static weight of exactly 9.36 kN. During operation, the total load on the spring is estimated to fluctuate up to 10.67 kN maximum. The spring is forged of a 95-point carbon steel and after heat treatment has the following minimum properties: S_{ut} = 1400 MPa, S_{yt} = 950 MPa, H_B = 399, and 32 percent reduction in area. Estimate the factor of safety if the spring is 18 mm thick.